Table X

SYNOPTICAL HISTORY OF GREEK GEOMETRY AND ASTRONOMY.

GREEK HISTORY: *GEOMETRY :* *ASTRONOMY :*

Thales (640-546 *B.C.), of Miletus.*

About 600 B.C., visited Egypt, and studied science there.

Hieronymus of Rhodes *(ap. Diog. Laer I,* 27) says, " He never had any teacher except during the time when he went to Egypt and associated with the priests." On his return from Egypt he founded the Ionian School of Astronomy and Philosophy.

Originated the equation and proportion, and was thus in a sense the originator of Algebra. Is recognised to have been the founder not only of Greek geometry, but also of Greek astronomy and philosophy. He also " founded," on a scientific basis, the geometry of the circle and of points and lines. (Proclus, *In primum Euclidis Elementorum Librum Commentarii*; Prof. G. J. Allman, " Greek Geometry from Thales to Euclid " ; Enc. Brit., Vol. 26, pp. 720-721.)

He taught " that the sun, moon, and stars are not mere spots on the heavenly vault, but solids ; that the moon derives her light from the sun, and that this fact explains her phases ; that an eclipse of the moon happens when the earth cuts off the sun's light from her." (Prof. G. Forbes' "Hist. Astron." p. 13). He also taught the sphericity of the earth, and the obliquity of the ecliptic. (Dr. F. R. Moulton's " Celestial Mechanics," p. 30 ; Miss A. Clerke in " Enc. Brit " Vol. 2, p. 809.)

Pythagoras (569-470 *B.C.), of Samos.*

Travelled widely in the East, visiting Chaldaea, and penetrating as far as the Ganges.

About 500 B.C. he visited Egypt, and studied science there. Returning from his travels, he founded a School of Astronomy and Philosophy in Sicily.

The Pythagorean doctrine of the immortality of the soul is clearly of Egyptian origin, whereas the connected Pythagorean doctrine of transmigration of the soul is certainly of a more easterly origin.

The Pythagorean idea of placing natural phenomena on a numerical basis, of associating numbers with conceptions and entities, is also clearly Egyptian in its origin. Thus of the ancient Egyptians Dr. Sprenger states "An idea, a period of time, or any remarkable occurrence, were frequently connected with ideal persons in mythology, and when any similarity existed, received the same appellation." (Vyse's " Pyds. and Temp, of Gizeh," Vol. II, Append.). Hence the importance of the following from Dr. A. S. Pringle-Pattison: " Impressed by the presence of numerical relations in every department of phenomena, Pythagoras and his early followers enunciated **the** doctrine that 'all things are numbers ' Numbers seemed to things in the whole of nature, and they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and number. (*Meta.* A. 986a.). Numbers, in other words, were conceived at that early stage of thought not as relations or qualities predicable of things, but as themselves constituting the substance or essence of the phenomena—the rational reality to which the appearances of sense are reducible." (Enc. Brit., Vol. 22, p. 699.) Pythagoras discovered or more probably derived from the Egyptians the mathematical proportions of the intervals of the diatonic scale.

Originated that branch of geometry associated with his name, and dealing chiefly with areas and solids. He is credited with a knowledge of certain properties of Conic Sections, and the discovery of the law of the three squares (Euclid I, 47) is attributed to him. Diogenes Laertius states that " it was Pythagoras who carried geometry to perfection, after Moeris (Amenemhat III of the XHth Egyptian Dynasty) had first found out the principles of the elements of that science.....,, and the part of the science to which Pythagoras applied himself above all others was arithmetic." Prof. Allman states "According to Aristoxenus, the musician, Pythagoras seems to have esteemed arithmetic above everything, and to have advanced it by diverting it from the service of commerce and by likening all things to numbers. Diogenes Laertius (viii, 13) reports on the same authority that Pythagoras was the first person who introduced measures and weights among the Greeks." In the system of Pythagoras " Ten" was a sacred number and the most perfect number. He was acquainted with arithmetical, geometrical, and harmonical proportion, and concerned himself with finding geometrical representations of numbers. He also elaborated the conceptions of the equation and proportion as "originated" by Thales.

" He taught that the earth both rotates and revolves, and that the comets as well as the planets move in orbits around the sun. He is credited with being the first to maintain that the same planet, Venus, is both evening and morning star at different times." (Moulton, P- 3I-)

Pythagoras " learned on his travels to recognise the obliquity of the ecliptic, and to regard the earth as a sphere freely poised in space. The tenet of its axial movement was held by many of his followers." (Miss A. M. Clerke, Enc. Brit., Vol. 2, p. 809.)

" Copernicus in the sixteenth century claimed Pythagoras as the founder of the (heliocentric) system which he, Copernicus, revived." (Forbes, p. 14.)

(Authorities as above, and Enc. Brit., Vol. 22, pp. 700-703.)

Authorities, however, differ as to whether the system of Pythagoras was truly heliocentric. Thus Dr. A. S. Pringle-Pattison (Enc. Brit., Vol. 22, pp. 689-700) states that the Pythagoreans conceived " the earth as a globe self-supported in empty space, revolving with other planets round a central luminary.... The Pythagoreans did not, however, put the sun in the centre of the system. That place was filled by the central fire."

Democritus (*circ.* 570-460 *B.C.).*

He studied astronomy for 5 years (or 7 years ?) in Egypt (Diodor., I, 98), and claimed to have been a disciple of the Egyptian priests and the Magi, having visited also Persia and Babylon (Clem. Str., i, p. 304). He knew of the obliquity of the ecliptic.

Anaxagoras (*born* 499 *B.C)*

... studied astronomy in Egypt. " He held that in a solar eclipse the moon hides the sun, and in a lunar eclipse the moon enters the earth's shadow." (Forbes, p. 14).

Eudoxus (408-355 *B.C.) of Cnidus.*

Visited Egypt with Plato.

His geometrical work comprised the establishing of expressions for the volumes of the pyramid, prism, cone, and cylinder.

In astronomy, he first suggested arbitrarily representing the apparent motions of the sun, moon, and planets as taking place upon revolving spheres ; the motion of each planet being resolved into its components, each component being given a separate revolving sphere. The hypothesis was not stated as an actual belief, but rather as a mathematical conception—failing any then more satisfactory conception—to permit of the formulating of rules and methods for making astronomical calculations. This system— the Eudoxian or " homocen-tric "—was elaborated by Callipus and Aristotle in the middle of the 4th century B.C. (about 350-330 B.C.)

Plato (429-350 *B.C.), the Athenian philosopher.*

Visited Egypt and Cyrene, In Egypt he conversed with the Egyptian priests. He was the pupil of Socrates, and was a follower of Pythagoras.

Plato touched upon astronomical and geometrical questions, only when these came within the scope of his system of philosophy. In no strict sense can he be termed a mathematician nor yet an astronomer. He, however, " proposed to astronomers *the problem of representing the courses of the planets by circular and uniform motions."* (Forbes, p. 17.)

Euclid *lived during the reign of Ptolemy I, king of Egypt* (323-285 *B.C.)*

He is said to have founded the school of mathematics at Alexandria.

Euclid's great geometrical work is " The Elements," contained in thirteen books, in which is laid down the fundamental basis of that branch of modern mathematics known as Euclidian geometry.

One work, Euclid's *Phaenomena*, is of an astronomical nature and deals with problems concerning the apparent motion of the celestial sphere.

It is generally admitted that few of the propositions, theorems, etc., in Euclid's *Elements* are original. Euclid merely compiled and arranged the hitherto unsystematized geometrical work of his predecessors. He placed the geometry of the line and the circle on a soundly logical basis and in a sequence that has had more influence upon modern method than authorities have taken the pains to note or admit.

Aristyllus *and* ***Timocharis*** *(circ.* 320-260 *B.C.)* of the school of Alexandria.

They observed at Alexandria, and constructed the first star-catalogue.

Aristarchus (320-250 *B.C.) of Santos.*

Studied astronomy at Alexandria.

He wrote a work on "Magnitudes and Distances" describing a *theoretically* sound method of determining the relative distances of the sun and moon. He correct!) ' determined the sun's diameter at half a degree, and according to Archimedes had formulated a heliocentric planetary system in advance of the more complicated heliocentric system of the Pythagoreans.

Archimedes *(circ.* 287-212 B.C.) *of Syracuse in Sicily.*

Studied mathematics at Alexandria.

His geometrical works comprise treatises on the sphere and cylinder, on the measurement of the circle (showing that the value of π is between 3) and 3$^{1}/_{7}$), on conoids and spheroids, on spirals, etc.

A work of an astronomical nature was his now lost work *On Sphere-making.* Professor F. R. Moulton (under the heading of " Dynamical Astronomy," states that " Archimedes is the author of the first sound ideas regarding mechanical laws. He stated correctly the principles of the lever and the meaning of the centre of gravity of a body .....It is a remarkable fact that no single important advance was made in the discovery of mechanical laws for nearly 2000 years after Archimedes, or until the time of Stevinus (1548-1620), who was the first, in 1586, to investigate the mechanics of the inclined plane, and of Galileo (1564-1642), who made the first important advance in Kinetics."

Eratosthenes (276-196 *B.C.)*

A Greek astronomer in charge of the library at Alexandria in the reign of Ptolemy III, Euergetes.

He determined (approximately correct) the value of the obliquity of the Ecliptic, and the circumference of the earth. His version of the Egyptian Dynastic Chronology contains periods derived from Genesis, from the true period of the Precession of the Equinoxes, and from the Pyramid base measure in common Egyptian cubits. (Refer Plate XVI and ¶¶ 94 and 102.)

Apollonius *of Perga*

Lived during the reigns of Ptolemy III Euergetes, and Ptolemy IV Philopater (B.C. 247-205). He studied mathematics at Alexandria.

Apollonius wrote the famous treatise on conic sections that earned for him the title, " the great geometer."

Inastronomy Apolloniusoriginated the working hypothesis of epicycles, which hypothesis formed the basis for all astronomical conceptions and observations from Ptolemy to Copernicus. The hypothesis of epicycles originated from the " homqcentric " system of Eudoxus, but was a considerable advance on the latter, from point of view of application to practical problems.

Hipparchus (190-120 B.C.)

Born at Nicaea in Bithynia. He settled at Rhodes and possibly later at Alexandria.

He founded the science of trigonometry, plane and spherical, and compiled the first table of chords.

He is said to have founded the science of observational astronomy. More accurately, we may say that he was the first of a long series of practical astronomers whose observations were placed on record.

He is similarly stated to have invented the planisphere, which, however, he borrowed from the Chaldaeans. Astronomical historians are now generally agreed that Hipparchus owed much of his observational data to the long series of observations that had been carried out by the Chaldaeaus for many centuries, if not for close on 2000 years, before his time. Thus Prof. Forbes states (p. 18) that " making use of Chaldaean eclipses, he was able to get an accurate value of the moon's mean motion." This is in fact stated by Ptolemy in his *Almagest.* (Refer Prof. Simon Newcomb's use of the data of Hipparchus and Ptolemy, in his " Researches on the motion of the Moon," published by U.S.A. Govt. Printing *Office,* 1878.)

Probably much of the Chaldaean data of Hipparchus was derived from the works of the Chaldsan priest of Bel, Berosus or Berossus (the Greek form of his name), who " appears to have compiled his works in the reign of Antiochus II,B.C. 261-46." (Brown's " Prim. Constell." Vol. II, p. 331.) As Mr. Brown states, " he (Berossus) also compiled various astronomical treatises, which have unfortunately been lost; they furnished material for Greek writers such as Diodoros, and the most important of them was a translation of what Prof. Sayce calls ' the standard astrological work of the Babylonians and Assyrians.'..... Opinions of BSrosos respecting the moon have been preserved by Plutarch, Stobaios, and Vitruvius, and the latter *(De Architect,* IX, iv, 7) states that he treated of the properties of the signs of the Zodiac, of the planets, and of the sun and moon ; and that he established a school of learning in the island of Kos." Kos or Cos, the modern Turkish Island of Stanko, is at the mouth of the Gulf of Halicarnassus (Asia Minor), and about 50 miles North-West of the Island of Rhodes where Hipparchus had settled.

Centuries before Hipparchus, the Chaldaeans, Egyptians, and Chinese knew of the " Precession of the Equinoxes." It is, however, claimed for Hipparchus that he discovered the " Precession" quite independently of the ancients from a comparison of his own observations and those of Timocharis at Alexandria. Syncellus in his " Chronographia " states that the " fabled period " of the Precession, amongst the Egyptians and Greeks, was a period of 25 Sothic Cycles of 1461 " years," or altogether 36,525 years. This gives a rate of 35-J-" of angle per year, and the rate determined by Hipparchus was estimated by him as not less than 36". As to whether the rate of Hipparchus was influenced by the rate of the " fabled period " noted by Syncellus, or that of Syncellus derived from Hipparchus it is for our further discussion to show.

Hipparchus was the first to observe and appreciate the elements of the orbit of the earth, (or rather the apparent orbit of the sun), and the orbit of the moon, and by many bold conceptions based on his own vast experience of celestial observation—conceptions that were vastly in advance of his times—he anticipated in many features the basal requirements of the modern astronomical Ephemeris. He compiled the first solar tables, and also compiled a catalogue of 1080 stars on a constellational basis borrowed from the Chaldaeans. His realization of the eccentricities of the orbits was a further great advance in geometrical astronomy. He, however, believed that all bodies revolved round the earth as centre.

Menelaus *of Alexandria*

...flourished towards the end of the 1st century B.C. His mathematical work considerably advanced the science of Spherical Trigonometry and Astronomy.

Ptolemy *(fl. circ.* 120-160 A.D.)

was a native Egyptian, famous not only for his classical treatment of mathematical, astronomical, and geographical problems, but also for his having preserved in his great astronomical work, the *Almagest,* astronomical and chronological data—containing observations and records of Hipparchus and the Chaldaeans—-that has enabled history to be placed on a scientifically accurate basis.

Ptolemy may be said to have done for the spherical geometry and trigonometry of Hipparchus and Menelaus what Euclid did for the work of the earlier geometers. He also combined and systematized the " eccentric" hypothesis of Hipparchus, and the " epi-cyclic '' hypothesis of Apollonius of Perga.